Analysis and Computation of Models

Applied and Comp Math 935

Section: G100

Term: 2010 Spring

Instructor: Ralf
Wittenberg

Discussion Topics: Nonlinear Science: Instability, Patterns, Chaos and Turbulence

Overview:
The world around us displays considerable regularity and structure in the midst of complexity. It is particularly fascinating that quite distinct physical, chemical or biological systems frequently display similar patterns and dynamics, which suggests the presence of some underlying unifying mechanisms responsible for these patterns. A goal of this course is to study how a mathematical description via ordinary or partial differential equations can capture many of these mechanisms.

Through the study of simplied model PDEs, and of systems such as convecting fluids and reacting and diffusing chemicals, we will investigate some of the mathematics and physics underlying patterns and complexity in space and time, and learn various analytical techniques used to study the interplay between instability and nonlinearity that can give rise to such rich behaviours. Beginning with low-dimensional dynamical systems, our explorations will lead us to linear and nonlinear stability analyses and the Turing bifurcation; hence to weakly nonlinear systems and their patterns and secondary instabilities, via multiple-scale analysis and amplitude equations; and further to strongly nonlinear systems. A further objective is to make some progress towards (partially) characterizing the phenomena of chaos, spatiotemporal chaos and turbulence. Our tools will include asymptotic methods and Matlab-based numerical explorations, but we will also aim for more precise mathematical statements when appropriate.

Grading: The grade will be based on
Homework (40%),
Midterm (30%),
Final Project (30%)

Required Texts: No
textbook required

Recommended Texts: There is no single text that is suitable for this course. Two books that cover much of this material are:
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Michael Cross and Henry Greenside, Pattern Formation and Dynamics in Nonequilibrium Systems
(Cambridge University Press, 2009),
* Paul Manneville, Instabilities, Chaos and Turbulence (Imperial College Press, 2004);
but we will on occasion take a more mathematical approach and/or refer to additional sources.

Materials/Supplies:

Prerequisite/Corequisite: A solid undergraduate-level familiarity with the analysis and numerical solution of ordinary and partial differential equations. Some exposure to dynamical systems, fluid dynamics and/or asymptotic methods will be helpful. The absolute minimal course prerequisites are thus Math 310 and Math 314, or equivalent. More generally, for a graduate course such as this, anyone who is interested in the course but who is concerned about prerequisites should discuss this with the instructor.

Notes:

THE INSTRUCTOR RESERVES THE RIGHT TO CHANGE ANY OF THE ABOVE

INFORMATION.

Students should be aware that they have certain rights to confidentiality concerning the return of course papers and the posting of marks. Please pay careful attention to the options discussed in class at the beginning of the semester.